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Comparison of heuristic procedures for an integrated model for production and distribution planning in an environment of shared resources

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Abstract

In this paper, we present a mathematical model which integrates tactical-operational production and distribution decisions in a shared resources environment. More precisely, we integrate lot sizing production and distribution decisions with vehicle routing decisions. We obtain a global multi-period multi-item multi-vehicle model where a capacity constraint models the link between production and distribution decisions. Three heuristics are presented in order to solve this global model. The first two ones are based on a decomposition approach of the global model in production and distribution submodels. The third heuristic offers a higher level of integration by taking into account transportation decisions in the production planning problem. Computational tests show that the performance of the heuristic depends on the amount of shared resources in the system, the type of customer demand but not on the weight of the production cost against the distribution cost. The three heuristics allow to tackle problems of larger size than an optimal solution approach.

Keywords: integrated model, production, distribution, shared resources.

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1 Introduction

In today's highly competitive world, managing and coordinating the supply chain is fundamental in order to reduce the overall cost while maintaining a high service level. Supply chain costs encompass not only the use of factory resources, i.e. the cost of transforming raw materials into finished products but also the cost of service to the customer, delivery costs and the costs associated with making the sale (e.g. marketing, client relationship ...). It is generally accepted that improvements in supply chain coordination increase efficiency more than individually optimizing functional areas. Indeed, interdepartmental integration allows better performance leading to increases in "service level, better management of inventory levels, higher forecast accuracy and greater customer and employee satisfaction" [16]. As stated by Kenneth B. Kahn et al.[16], interdepartmental integration can be defined as a mix of interaction and collaboration factors. The importance of each of those attributes in the mix will depend on the managerial situation (e.g. stable product lines versus product launches...). Interaction corresponds to information exchanges between departments through meetings or other similar activities, whereas collaboration places the emphasis on a strategic alignment of the objectives of different departments (e.g. shared vision, collective goals and joint rewards ...).

Nowadays, with the development of more precise information and technological tools, it is possible to develop integrated decision making tools. Those tools are based on global optimization methods where the boundaries between problems, tools and decision levels are eliminated.

Various types of coordination in the supply chain can be achieved at various levels (material procurement, production and distribution). For example, coordination between warehouse and inventory decisions has shown to be more efficient to handle warehouses with limited capacity than individually optimizing those areas [22]. From now on, we will concentrate our analysis on production and distribution activities.

Production and distribution activities can be handled independently using inventories as buffers between them, but this leads to important holding costs and longer lead times in the supply chain. Reducing inventories and lead times can be achieved by coordinating production and distribution decisions. Management decision tools based on modeling, optimization and simulation methods, where integration of decisions is an important component, help in coordinating those functional areas.

The integration of production and distribution decision making can be achieved at all levels namely strategic, tactical and operational. Strategic production and distribution decisions include issues of design and decisions in the supply chain such as location, plant capacity and transportation channels. At the tactical level, production and distribution decisions deal with questions such as how much to produce, how much to ship in a time period, how long the production cycle / distribution cycle should be, how much inventory to keep, ... Finally, operational production and distribution decisions concern problems of detailed scheduling: when and on which machine to process a job, when and by which vehicle to deliver a job, which route should the vehicle take ... We are interested in the integration of tactical and operational production and distribution decisions. More precisely, we have concentrated our work in three areas: at the tactical level lot sizing decisions in both production and distribution and at operational level the vehicle routing problem.

Our aim in this paper is to analyze the benefits of coordination (sharing of information) between the production and the distribution department when tangible resources are shared between those two functional areas.

To understand the importance of shared resources management and coordination between production and distribution departments, let us examine the gas filling industry. The company we consider specializes in industrial and medical gases and related services. It supplies oxygen, hydrogen and many other gases and services to a large number of customers that range from industry to hospitals, oil refineries and aerospace facilities. The company distributes its gas cylinders between its different filling centers, distribution centers and customers. These cylinders are an extremely practical method of supplying gas and are an extremely critical resource, the use of which creates many difficulties in production and distribution planning. A cylinder is defined by the nature of its contents (compressed gas, refrigerated liquid, etc.), the type of gas (O_2, N_2, CO_2, H_2, etc.), the gas purity and the size (volume and pressure). Cylinders can be supplied individually or in sets of 9 or 18 bundled together and emptied as a unit. Inventory management should provide a controlled access to cylinders and should allow for the return of empty gas containers as well as a precise tracking of cylinders to enable the supply of customers again when needed. Regulatory checks and retesting of cylinders have to be handled before any reuse. All these constraints makes the management of these cylinders a complex task involving different departments.

In the previous example, large quantities of shared resources need to be kept in inventories when there is no coordination between the production and distribution departments in order to have fluidity in the supply chain. When those shared resources are expensive, it can lead to high holding costs. Consequently, our aim is to develop a global model at the tactical and operational level to optimize the use of shared resources in production and distribution department. We also propose three different heuristics to solve this integrated model. The performance of our heuristics is evaluated by realizing computational tests. Finally, the advantages and disadvantages of our heuristics are highlighted.

In the second section, we make a brief review of the literature available on the subject. The third section introduces the model formulation and the various assumptions made. A description of the various heuristics used to solve the integrated model follows in Section 4 and the computational results are presented in Section 5.

2 Tactical and operational production and distribution models

In this section, we survey the literature dealing with the issues of coordination of production and distribution. As we are only interested in tactical and operational decisions in those fields, we concentrate our survey on the most important articles in that field. More precisely, we focus, in the first part of this review, on articles which are interesting due to the type of decision considered and the managerial environment involved. In the second part of this review, we cite articles where authors have concentrated on the development of good solution methodology to solve this coordination problem.

The coordination of production and distribution activities at the operational and tactical level has been sparsely analyzed in the literature. Chandra et al. [10] consider the coordination of production and distribution scheduling (VRP problem). They study a 2-echelon, multi-product, multi-period and multi-retailer system with one plant and deterministic demand. They propose two solution methodologies to solve this global model. The first one is based on a decomposition approach of the global model in production and distribution submodels. The second methodology consist in searching for cost reduction in the plan found by the first methodology. According to their computational results, savings of 3% to 20% can be achieved by coordinating those two functional areas and the value of integration increases with the length of the planning horizon and the number of products and customers.

Barbarosoglu et al. [3] study the potential benefit of coordinating production and distribution lot sizing decisions (no routing decisions are considered). They analyze a 3-echelon system, multi-product, multi-depot, multi-period and multi-customer, with deterministic demand. No production and distribution lead times are considered and no capacity limits are imposed on the inventories. They proposed to solve this global model by Lagrangian relaxation and subgradient optimization. This allows information to flow between those two models while keeping the advantage of a hierarchical structure. In addition, the distribution submodel is solved by the mean of a forward heuristic. The authors report that their algorithm provides good bounds in short CPU times even for large instances.

Fumero et al. [14] analyze the integration of capacity management, inventory allocation and vehicle routing decisions. They consider a 2-echelon, multi-customer, multi-period system with one plant and deterministic demand. No lead times are considered for the transportation of items from the plant to the customers. Their solution method is based on Lagrangean relaxation. Computational tests show that the value of coordination increases with the number of products and customers, with the available capacity for production and the fleet and with the number of time periods.

Ozdamar et al. [19] investigate a two-stage system composed of a factory and remote warehouses. No routing issues are considered in this problem. They propose a monolithic problem which is solved at different levels of aggregation based on a hierarchical structure. Backorders are allowed. An iterative solution methodology is proposed and tested on a database of detergent products.

Mohamed et al. [18] want to integrate the tactical production and distribution decisions for a multi-national company over a finite planning horizon considering a two-stage system composed of facilities and customers dispersed around the world. The issues considered are the location of product manufacturing, the assignment of markets to facilities and the influence of inflation and exchange rates on those decisions. Capacity at each facility is considered as a decision variable. Capacity can change from one period to another and this modification involves a capacity changing cost which is included in the objective function. No routing decisions are involved. Inventories are available at the factories and demands are deterministic and must be satisfied in a JIT way.

Dhaenens-Flipo and Finke [12] study a multi-facility, multi-product, multi-period industrial problem. They want to coordinate the distribution and the production function in a 3-echelon system composed of facilities (each with parallel production lines), warehouses and customers. Storage is allowed at the plants or at the warehouses and not at the customers. Customers' demands must be satisfied in a JIT manner. They consider setup cost, fixed and variable transportation costs. They formulate the problem as a network flow problem. Numerical experiments show that this model can solve large real-life industrial problems in reasonable computing time.

There has been some recent articles published on the Production, Inventory, and Distribution Routing Problem (PIDRP) where the aim of the authors was to develop a good solution methodology to solve this NP-hard problem. The PIDRP considers decisions of production and distribution lot sizing level combined with vehicle routing decisions. In general terms, we can describe PIDRP as a single item problem involving one plant with multiple customers, deterministic demand, a fleet of homogeneous capacitated vehicle.

Lei et al. [17] were the first to propose a MIP formulation for the PIDRP. They considered a variant of the PIDRP where there are multiple plants and a heterogeneous fleet of vehicles. They proposed a solution methodology based on a two phase approach where, in the first phase, routing decisions are limited to direct shipments. In the second phase, consolidation of less than truckload shipments found in the first phase is investigated.

Boudia et al. [7], [8], [9] proposed a MIP formulation somewhat similar to the previous authors and proposed to solve this model by the mean of different solution methodologies: a memetic algorithm with dynamic population management (MA|PM), a reactive GRASP and two improved mechanisms based on a reactive mechanism and a path relinking methodology and finally a combined greedy heuristics with a local search procedure. Computational results confirm the importance of integrating production and distribution decisions and their solution methodologies allow to tackle problem of reasonable size (up to 200 customers and 20 time periods) in short computational time. Moreover, their metaheuristic (MA|PM) has proved to be more efficient to tackle coordination of decisions at production and distribution level than the other heuristics that they have proposed.

Bard et al. [5] developed, based on the same MIP formulation, a solution methodology based on tabu search algorithm followed by a path relinking method to improve the final solution found. The novelty of the method is the elaboration of an allocation model used to find good starting feasible solutions for the tabu search procedure. Results were promising with improvements ranging from 10 to 20% compared to the result obtained with Boudia et al. [7] GRASP solution methodology but at a computational time cost. Some authors have investigated exact solution methodologies. This was the case of Bard et al. [4, 6] who investigated a heuristic implementation of a branch-and-price algorithm. Computational tests show that they were able to solve instances with 50 customers and 8 time periods within 1 hour. This performance cannot be achieved by CPLEX or standard branch-and-price alone.

Finally, Ruokoski et al. [21] proposed to solve the production-routing problem optimally by using a mixed integer linear programming formulation and several strong reformulations. Compared to the other authors, they consider the restricted environment of a single uncapacitated vehicle. In addition, they proposed two families of valid inequalities: 2-matching and generalized comb inequalities. Those reformulations combined with the valid inequalities are embedded in a branch-and-cut procedure and used to solve this coordination problem. Computational results show that their method allows to solve instances of up to 80 customers and 8 time periods within a two-hour time limit. They compare their methodology with traditional decomposition methods and a heuristic algorithm.

Our contribution differs from previous work because we analyze the integration of tactical/operational production and distribution decisions in a particular business environment where resources are shared between the two departments. Our aim is to analyze how the integration of those decisions helps in managing those shared resources.

3 Problem description and mathematical formulation

In this section, we propose a definition of shared resources in order to highlight the uniqueness of our analysis. Then we describe the assumptions we make on the environment and we propose a mathematical formulation of the problem as a mixed-integer program.

3.1 Definition of shared resources

Resources include "assets, capabilities, organizational processes, firm attributes, information, knowledge, etc. controlled by a firm which enables it to conceive and implement strategies that improve its efficiency and effectiveness" [11].

In general, resources can be classified as tangible resources, such as inventory, manufacturing resources (machinery, installation, plant, equipment etc.), logistics and transportation; or intangible resources such as information, technological innovation, human resources, intellectual property, development of new production processes and models, customer relationships, relationship between supply chain members, etc.

We are interested in managing resources in the supply chain that are shared among independent decision makers in order to improve its overall efficiency and effectiveness.

Independent decision makers can be of two types: inter firm partners or intra firm partners. In the first case, inter firm relationships can exist in a horizontal supply chain such as one company supplying components to another or in a vertical supply chain with retailers, distributors, and manufacturers. In the second case, intra firm partners can be different business units like production and distribution departments or procurement and production departments. Depending on the different types of resources to be shared, decision areas range from the operational decisions, for example dealing with inventory, capacity allocation, transportation decisions through the tactical decisions, for example, information sharing, negotiating contracts; to decisions taken at a strategic level, for example, investment decisions, facility locations, plant capacity, etc.

The aim of sharing resources between decision makers is to improve the efficiency and effectiveness across the entire supply chain. By this, we mean that the different parts need to benefit from the sharing. We need to assess the results of sharing resources using cost analysis and profit evaluation. The allocation of the collaboration surplus must lead to a more profitable situation for all actors of the collaboration ex post.

3.2 Problem description and assumptions

We have one manufacturing site which is delivering finished products to several customers. We are in a multi-item, multi-period, multi-vehicle, capacitated environment. There exist different types of product which are composed of a content (what is consumed by the customers) and a packaging. The packaging is the shared resource between the manufacturing unit and the customer. Indeed, without the packagings, the production process can not take place and the finished products can not be delivered to and consumed by the customers. Each packaging has the same volume. For the routing decisions, we have several types of vehicles with given capacities and setup costs. The delivery of the finished product and of the packaging is done simultaneously. In addition, the amount of packagings picked up at a customer site must be equal to the amount of finished product delivered. This assumption follows what is generally done in practice in business life in order to have a traceability of the packagings. Moreover, the order of a customer can not be split across different vehicles. Each customer is defined by a demand which is deterministic and a geographical position defined by Euclidean coordinates. We have two types of inventories in the system:

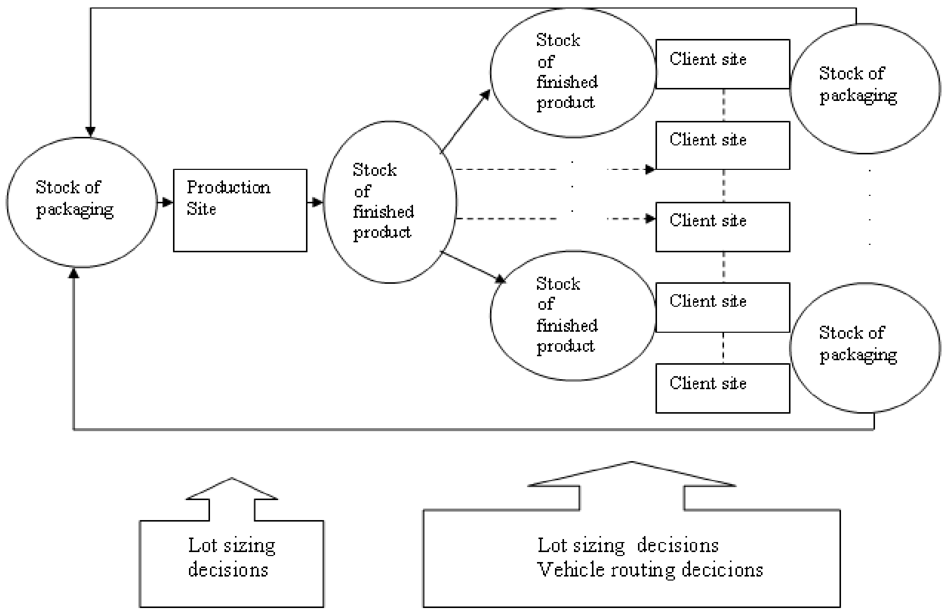

Figure 1: The problem under consideration

packaging inventories and finished product inventories. Those inventories are at the manufacturing site and at the various customer locations. We suppose that it takes a certain number of days for the customer to turn the finished product in a packaging. This duration is called the stock rotation. This amount of time is dependent on the customer consumption rate and on the type of product. The production and distribution decisions are planned over an horizon of a week.

The problem is represented graphically in Figure 1.

3.3 Mathematical formulation

In this section, we propose a mathematical model which integrates the production and distribution lot sizing problem with the vehicle routing problem in a multi-item, multi-period, multi-vehicle and capacitated shared resources environment. Therefore, we take into account in our objective function traditional production lot sizing costs (setup, production and storage costs), distribution lot sizing costs (setup and storage costs) and vehicle routing costs (traveling costs based on distances). The constraints of our model include the traditional production and distribution lot sizing constraints with the particularity that the production capacity will be limited by the amount of shared resources available at the production site. This implies a direct link between the production and the distribution decisions. In addition, we also have the traditional vehicle routing constraints.

The indices used are $i : 1, ..., I$ to denote a type of product and $k, m : 0, ..., K$ to denote a customer with the index value 0 denoting the manufacturing site. Vehicles are identified through the index $j : 1, ..., J$ and the time period through the index $t : 1, ..., T$.

We describe hereafter the data and variables used in the model. For each element, we give the units of measure between brackets.

Data:

- $D_{i,k,t}$= the demand of customer k for product i in period t[units]
- $length_{k,m}$= the Euclidean distance between a customer k and a customer m [km]
- $\delta_{i,k}$= the stock rotation of a product i for each client k [days]
- CV_j = the capacity of vehicle j [units]
- PC = production cost [€/unit]
- SC= production setup cost [€/setup]
- TC= cost of transportation [€/km]
- SCT_j = transportation setup cost for each vehicle j [€/setup]
- HFC = holding cost of finished product at the customer site [€/unit]
- HPC = holding cost of shared resources at the customer site [€/unit]
- HFU = holding cost of finished product at the manufacturing unit [€/unit]
- HPU = holding cost of shared resources at the manufacturing unit [€/unit]

Variables:

- $x_{i,t}$ = amount of finished product i produced in period t [units]
- $fu_{i,t}$= amount of finished product i available in stock at the manufacturing unit at the end of period t [units]
- $pu_{i,t}$= amount of shared resources of product i available in stock at the manufacturing unit at the end of period t [units]
- $z_{i,k,j,t}$= amount of finished product i delivered to customer k by the vehicle j in period t [units]
- $w_{i,k,j,t}$= amount of shared resources of product i transported from customer k by the vehicle j in period t to the manufacturing unit [units]

- $fc_{i,k,t}$= amount of finished product i available in the stock of client k in period t [units]
- $pc_{i,k,t}$= amount of shared resources of product i available in the stock of client k in period t [units]

$$y_{i,t} = \begin{cases} 1 & \text{if the production of product } i \\ & \text{is started in period } t \\ 0 & \text{otherwise} \end{cases}$$

$$p_{k,j,t} = \begin{cases} 1 & \text{if the client } k \text{ is served by} \\ & \text{the vehicle } j \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$$

$$l_{j,t} = \begin{cases} 1 & \text{if vehicle } j \text{ is used in period } t \\ 0 & \text{otherwise} \end{cases}$$

$$f_{j,k,m,t} = \begin{cases} 1 & \text{if vehicle } j \text{ travels from customer } k \\ & \text{to customer } m \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$$

The objective function is as follows:

$$\min \sum_{i=1}^{I}\sum_{t=1}^{T} PC * x_{i,t} + \sum_{i=1}^{I}\sum_{t=1}^{T} SC * y_{i,t} \tag{1}$$

$$+ \sum_{i=1}^{I}\sum_{t=1}^{T} HFU * fu_{i,t} + \sum_{i=1}^{I}\sum_{t=1}^{T} HPU * pu_{i,t} \tag{2}$$

$$+ \sum_{j=1}^{J}\sum_{k=0}^{K}\sum_{m=0}^{K}\sum_{t=1}^{T} TC * length_{k,m} * f_{j,k,m,t} \tag{3}$$

$$+ \sum_{j=1}^{J}\sum_{t=1}^{T} SCT_j * l_{j,t} \tag{4}$$

$$+ \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} HFC * fc_{i,k,t} \tag{5}$$

$$+ \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} HPC * pc_{i,k,t} \tag{6}$$

The objective function is composed of different terms of cost related to the production lot sizing problem and to the transportation problem. The lot-sizing decisions costs concern production cost, production setup cost (1) and holding cost at the manufacturing site (2). The transportation decision costs (3) concern the cost of transportation of empty containers from customers to the manufacturing unit and the cost of transportation of finished product from the manufacturing unit to the customers. An additional transportation setup cost is incurred each time a vehicle is leaving the manufacturing site (4). There is also holding cost of empty containers and finished product at the different customer sites (5),(6).

Constraints:

$$x_{i,t} + fu_{i,t-1} = \sum_{k=1}^{K}\sum_{j=1}^{J} z_{i,k,j,t} + fu_{i,t} \qquad \forall i,t \tag{7}$$

$$x_{i,t} \leq (\sum_{k=1}^{K}\sum_{n=t}^{T} D_{i,k,n}) \times y_{i,t} \qquad \forall i,t \tag{8}$$

$$pu_{i,t-1} + \sum_{k=1}^{K}\sum_{j=1}^{J} w_{i,k,j,t-1} = pu_{i,t} + x_{i,t} \qquad \forall i,t \tag{9}$$

$$\sum_{j=1}^{J} z_{i,k,j,t} + fc_{i,k,t-1} = D_{i,k,t} + fc_{i,k,t} \qquad \forall i,k,t \tag{10}$$

$$pc_{i,k,t-1} + D_{i,k,t-\delta_{i,k}} = pc_{i,k,t} + \sum_{j=1}^{J} w_{i,k,j,t} \qquad \forall i,k,t \tag{11}$$

$$\sum_{i=1}^{I}\sum_{k=1}^{K} z_{i,k,j,t} \leq CV_j \qquad \forall j,t \tag{12}$$

$$\sum_{i=1}^{I} z_{i,k,j,t} \leq CV_j \times p_{k,j,t} \qquad \forall k,j,t \tag{13}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J} w_{i,k,j,t} = \sum_{i=1}^{I}\sum_{j=1}^{J} z_{i,k,j,t} \qquad \forall k,t \tag{14}$$

$$\sum_{j=1}^{J} p_{k,j,t} \leq 1 \qquad \forall k,t \tag{15}$$

$$p_{k,j,t} \leq l_{j,t} \qquad \forall k,j,t \tag{16}$$

$$\sum_{m=0,k\neq m}^{K} f_{j,k,m,t} - \sum_{m=0,k\neq m}^{K} f_{j,m,k,t} = 0 \qquad \forall k,t,j \qquad (17)$$

$$\sum_{m=1}^{K} f_{j,0,m,t} = l_{j,t} \qquad \forall j,t \qquad (18)$$

$$\sum_{m=0}^{K} f_{j,k,m,t} = p_{k,j,t} \qquad \forall j,k,t \qquad (19)$$

$$\sum_{k,m\in S:k\neq m} f_{j,k,m,t} \leq |S| - 1 \qquad \forall j,t,S \subset \{1,\ldots,K\} \qquad (20)$$

$$x_{i,t}, fu_{i,t}, pu_{i,t} \geq 0 \qquad \forall i,t \qquad (21)$$

$$z_{i,k,j,t}, w_{i,k,j,t} \geq 0 \qquad \forall i,k,j,t \qquad (22)$$

$$fc_{i,k,t}, pc_{i,k,t} \geq 0 \qquad \forall i,k,t \qquad (23)$$

$$y_{i,t}, p_{k,j,t}, l_{j,t}, f_{i,k,m,t} \in \{0,1\} \qquad \forall i,k,m,j,t \qquad (24)$$

Constraints (7) and (9) are flow balance constraints at the manufacturing site for the storage of finished product and at the manufacturing site for the storage of empty containers. Note that the later also limits the capacity at the manufacturing site because the production can only take place if empty containers are available in stock. Constraints (10) are the flow balance constraints of finished products at the client site while constraints (11) are the flow balance constraints of empty containers at the client site. Constraints (8) are production setup constraints. Constraints (12)-(20) are transportation constraints. Constraints (12) are the vehicle capacity constraints for finished products. Constraints (13) assign vehicles to clients. Constraints (14) ensure that the collecting of empty containers at each customer site equals the quantity of finished products delivered. Constraints (15) force each client to be served by one and only one vehicle: orders can not be divided and delivered by different vehicles. Constraints (16) define the setup variable for each period and for each vehicle. Constraints (17)-(20) model the vehicle routing part of the problem.

4 Valid inequalities

Our aim in this section is to tighten the initial formulation presented in Section 3.3 by adding valid inequalities.

Hereafter, we present three high level relaxations of our model in order to derive valid inequalities which are going to be added a priori to the formulation (before the optimization starts). Those relaxations are lot-sizing relaxations [20].

The first valid inequality is defined by the submodel composed of constraint (7), (8) and (10). By combining the first two constraints, we obtain the following relaxation for each product and for each period:

$$x_{i,t} + fu_{i,t-1} + \sum_{k=1}^{K} fc_{i,k,t-1} = \sum_{k=1}^{K} D_{i,k,t} + fu_{i,t} + \sum_{k=1}^{K} fc_{i,k,t} \qquad \forall i,t \tag{25}$$

For ease of presentation, we use the concept of echelon stock [20] and define variable $e_{i,t}$ which replace variables $fu_{i,t} + \sum_{k=1}^{K} fc_{i,k,t}$. Constraints (25) become:

$$x_{i,t} + e_{i,t-1} = \sum_{k=1}^{K} D_{i,k,t} + e_{i,t} \qquad \forall i,t \tag{26}$$

Consequently, constraints (26) and the production setup constraints (8) define an uncapacitated single item lot sizing relaxation. Therefore, we can deduce the following valid inequality:

$$e_{i,t-1} \geq \sum_{k=1}^{K} \sum_{m=t}^{o} D_{i,k,m} \times (1 - \sum_{n=t}^{m} y_{i,n}) \qquad \forall i, t \in T, o \in T, o \geq t \tag{27}$$

The second valid inequality is based on the submodel composed of constraints (10) and (13). By defining the following new variables and parameters $CV' = \sum_{j=1}^{J} CV_j$ and $z'_{k,t} = \sum_{i=1}^{I} \sum_{j=1}^{J} z_{i,k,j,t}$, $D'_{k,t} = \sum_{i=1}^{I} D_{i,k,t}$, $fc'_{k,t} = \sum_{i=1}^{I} fc_{i,k,t}$, we can derive a relaxation of constraints (10) for each customer at each period:

$$z'_{k,t} + fc'_{k,t-1} = D'_{k,t} + fc'_{k,t} \qquad \forall k,t \tag{28}$$

As $\sum_{j=1}^{J} p_{k,j,t} \leq 1 \ \forall k,t$, we can rewrite constraint (13) as:

$$z'_{k,t} \leq CV' \sum_{j=1}^{J} p_{k,j,t} \qquad \forall k,t \tag{29}$$

Therefore, constraints (28) and (29) define a capacitated lot sizing relaxation for each customer at each period. The following valid inequality can be derived:

$$\sum_{i=1}^{I} fc_{i,k,t-1} \geq \sum_{i=1}^{I}\sum_{m=t}^{o} D_{i,k,m} \times (1 - \sum_{j=1}^{J}\sum_{n=t}^{m} p_{k,j,n}) \qquad \forall k, t \in T, o \in T, o \geq t \tag{30}$$

The third inequality is derived from the submodel composed of constraints (11), (13) and (14). We define the following additional new variables: $pc'_{k,t} = \sum_{i=1}^{I} pc_{i,k,t}$, , $w'_{k,t} = \sum_{i=1}^{I}\sum_{j=1}^{J} w_{i,k,j,t}$

With the use of those new variables, a relaxation of the submodel (11) and (14) can be derived for each customer and for each period:

$$pc'_{k,t-1} + D'_{k,t-\delta'_k} = pc'_{k,t} + w'_{k,t} \qquad \forall k, t$$
$$w'_{k,t} = z'_{k,t} \qquad \forall k, t$$

Combining the first two constraints, we get:

$$pc'_{k,t-1} + D'_{k,t-\delta'_k} = pc'_{k,t} + z'_{kt} \qquad \forall k, t \tag{31}$$

Constraints (31) and (29) define a capacitated lot sizing relaxation for each customer at each period. Therefore, we can derive the following valid inequality:

$$\sum_{i=1}^{I} pc_{i,k,t} \geq \sum_{m=\delta_{i,k}+1}^{o}\sum_{i=1}^{I} D_{i,k,m-\delta_{i,k}} \times (1 - \sum_{n=m}^{T}\sum_{j=1}^{J} p_{k,j,n}) \quad \forall k \in K, t, o \in T, o \geq m, o \leq t \tag{32}$$

Due to the limited number of those valid inequalities, we will add them a priori to our initial formulation (before the optimization starts) in order to tighten the solution space and not by the mean of a separation algorithm.

Computational tests on a reduce size data set [1] show that the gap at the root note can be reduces by 26% and the total branch-and-bound computational time by 45% by including these valid inequalities.

5 Heuristic approaches

The model presented in Section 3.3 is a linear mixed integer model, multi-item, multi-period. Our aim is to solve this model at the operational level.

[1] The dataset used is the same as the one used in section 6.2.1

Therefore, we want a good solution in short computational time. Even though the valid inequalities added to the model allow to tighten the model and consequently improve the lower bound, a branch-and-bound approach does not allow to fulfill this requirement [23] (this is also confirmed by our computational results). Consequently, our aim is to find a procedure to solve the problem heuristically.

We propose in this section three different heuristics. Two of them are based on a decomposition approach of the global model into submodels whereas the third one is a more integrated heuristic. The later does not rely on a decomposition approach but tries to solve the global model as a whole.

The main advantage of using a decomposition approach to solve this global model is to reduce the complexity of the model and therefore to enable to solve larger instances of the model. Nevertheless, this simplification implies that the coordination between production and distribution decisions is reduced and that infeasibility problems can occur. In order to avoid infeasibility problems resulting from our decomposition approach, we introduce an additional variable ($puadd_i$) which represents the additional packaging of type i needed in the system. This additional shared resources can be rented at an expensive cost.

Our integrated heuristic allows to estimate the advantage of using a more integrated approach in solving our global model, with the drawback of being more computationally challenging.

5.1 Sequential production-transportation heuristic

This heuristic is based on a decomposition procedure where the global production and transportation model is divided in an uncapacitated lot sizing model and a distribution model for the shared resources and the finished product. This methodology mimics what is commonly done in business in order to reduce the complexity of the production and distribution model. Those production and transportation models are solved sequentially: first the production model is solved then the transportation model is solved based on the solution of the production planning model. Consequently, the lot sizing model is solved without taking into account the shared resources capacity restriction involved by the distribution decisions. Once the production planning decisions are fixed, the transportation model is solved based on the production planning restrictions. Those transportation decisions are approximated by a generalized assignment heuristic [13] where a general assignment problem is solved followed by a traveling salesman problem. Those two submodels are presented in the following section.

5.1.1 Production planning decisions

The production planning decisions consist in choosing, for the manufacturing sites, the production level, setup run and the level of finish product stocks for each product and at each time period. This is formulated as an uncapacitated lot sizing model where the demand of product i of client k in period t takes into account the initial stock available at customer site.

$$\min \sum_{i=1}^{I}\sum_{t=1}^{T} PC * x_{i,t} + \sum_{i=1}^{I}\sum_{t=1}^{T} SC * y_{i,t} + \sum_{i=1}^{I}\sum_{t=1}^{T} HFU * fu_{i,t} \tag{33}$$

subject to

$$x_{i,t} + fu_{i,t-1} = \sum_{k=1}^{K} D'_{i,k,t} + fu_{i,t} \qquad \forall i,t \tag{34}$$

$$x_{i,t} \leq (\sum_{k=1}^{K}\sum_{o=t}^{T} D'_{i,k,o}) \times y_{i,t} \qquad \forall i,t \tag{35}$$

$$x_{i,t}, fu_{i,t} \geq 0 \qquad \forall i,t \tag{36}$$

$$y_{i,t} \in \{0,1\} \qquad \forall i,k,m,j,t \tag{37}$$

where

$$D'_{i,k,t} = \max\left(0, \min\left(\sum_{e=1}^{t} D_{i,k,e} - fc_{i,k,0}, D_{i,k,t}\right)\right) \qquad \forall i,k,t$$

Each single item subproblem has Wagner-Whitin costs [20] as the production cost does not change from one period to the other and can be solved in $O(T)$ time.

5.1.2 Transportation decisions for shared resources and finished product

The transportation decisions consist in choosing the amount of finished product and of shared resources to deliver and pick up at each customer site in each period in order to satisfy the demand of customers and to ensure enough capacity at the manufacturing site (enough shared resources). Those decisions have to be supported by computing the optimal route for each vehicle at each period. This transportation model is therefore composed of distribution decisions and of vehicle routing decisions where the variables $x_{i,t}$ are fixed at their values $\overline{x_{i,t}}$ computed at the previous step. We have to solve the global model (1)-(24) with the valid inequalities (27), (30) and (32) where we fix the production level ($\overline{x_{i,t}}$) in constraints (7) and (9) and the setup variables ($\overline{y_{i,t}}$) in constraints (27) as follows:

$$\overline{x_{i,t}} + fu_{i,t-1} = \sum_{k=1}^{K}\sum_{j=1}^{J} z_{i,k,j,t} + fu_{i,t} \qquad \forall i,t \tag{38}$$

$$pu_{i,t-1} + puadd_i + \sum_{k=1}^{K}\sum_{j=1}^{J} w_{i,k,j,t-1} = pu_{i,t} + \overline{x_{i,t}} \qquad \forall i,t \tag{39}$$

$$e_{i,t-1} \geq \sum_{k=1}^{K}\sum_{m=t}^{o} D_{i,k,m} \times (1 - \sum_{n=t}^{m} \overline{y_{i,n}}) \qquad \forall i, t \in T, o \in T, o \geq t \tag{40}$$

5.1.3 The Generalized Assignment Heuristic

Our transportation model, presented in section 5.1.2, is complex to solve due to the routing decisions (17)-(20). Therefore, in order to solve this transportation model for the shared resources and the finished product, we adapt the heuristic developed by Fisher and Jaikumar [13] for the Vehicle Routing Problem (VRP), to our particular situation. This heuristic, according to results reported by Gendreau et al. [15], is very efficient for the VRP.

The version of VRP which is tackled by Fisher and Jaikumar considers a set of customers with known demand levels and a set of vehicles with fixed capacities. Those vehicles must be loaded at a depot, visit customers and return to the depot. Decisions such as which vehicle will serve which demand with which route in order to minimize delivery cost are answered. In order to solve the VRP, they propose to use a Generalized Assignment Heuristic (GAH) composed of two phases: First a Generalized Assignment Problem (GAP) is solved to determine the assignment of customers to vehicles based on an approximation of the traveling costs. Secondly, a Traveling Salesman Problem (TSP) is solved to determine the optimal route for each vehicle. Hereafter, we give more details on each of the GAH phase.

1. The Generalized Assignment Problem

   As stated above, in the first part of Fisher and Jaikumar heuristic, a GAP is solved: customers are assigned to vehicle based on a approximation of the traveling costs. This approximation is based on the definition of a seed location for each vehicle. The seed location corresponds more or less to the area of operation of the vehicle. The cost of assigning a customer to a vehicle is based on the distance between the customer and the seed location for that vehicle (Euclidean distance). Fisher and Jaikumar determine the seed location so that the total demand of the customers in the region covered by a vehicle corresponds, more or less, to the vehicle capacity.

Our problem differs from Fisher and Jaikumar's basic vehicle routing problem because the quantity transported by the vehicle is not given. The choice of the seed locations is a decision to optimize. Therefore, we have adapted their heuristic to our particular problem.

We have decided to fix the possible location of the seeds depending on the location of the customers. Consequently, a grid is formed over the space delimited by the customers. Each intersection in the grid is a potential seed location. A first decision to optimize is to decide which of those seeds will be used by which vehicle. Then a second decision is to assign each customer to a vehicle. The additional transportation cost incurred if a customer is allocated to a vehicle can be estimated. This Generalized Assignment Problem is solved using a MIP solver with a formulation discussed below.

2. The Traveling Salesman Problem

   Once the Generalized Assignment Problem is solved, we know the optimal allocation of customers to vehicles and of vehicles to seeds. Based on this optimal allocation, a traveling salesman problem is solved for each vehicle at each period. Those TSPs are solved using the publicly available TSP solver Concorde [1, 2].

The transportation problem presented in Section 5.1.2 is solved using the adapted general assignment heuristic presented above. For this purpose, we define new variables for the seed locations which are referred by index $s : 1, ..., S$, and new transportation costs.

The new variables are:

$$\beta_{k,s,j,t} = \begin{cases} 1 & \text{if customer } k \text{ is assigned to} \\ & \text{seed } s \text{ and vehicle } j \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$$

$$q_{j,s,t} = \begin{cases} 1 & \text{if vehicle } j \text{ is assigned to} \\ & \text{seed } s \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$$

The approximate transportation cost for the GAP formulation corresponds to the additional cost incurred when an extra customer is assigned to the route of a vehicle traveling from the depot location 0 to a seed s. Therefore we can define $\overline{length}_{k,s}$ as the additional distance traveled by a vehicle if an extra customer k is serviced by this vehicle.

$$\overline{length}_{k,s} = length_{0,k} + length_{k,s} - length_{0,S}$$

The new problem is:

$$\begin{aligned}
\min \; & \sum_{k=1}^{K}\sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=1}^{T} TC * \overline{length}_{k,s} * \beta_{s,k,j,t} \\
& + (2) + (4) + (5) + (6) \\
& + 2 * TC * \sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=1}^{T} (length_{0,s} * q_{j,s,t})
\end{aligned}$$

subject to

$$\begin{aligned}
& (40), (30), (32), (38), (39), (10), (11), (12), (13), (14), (15), (16), (22), (23) \\
& p_{k,j,t} = \sum_{s=1}^{S} \beta_{k,s,j,t} && \forall k, j, t \\
& \sum_{s=1}^{S}\sum_{j=1}^{J} \beta_{k,s,j,t} \leq 1 && \forall k, t \\
& \sum_{s=1}^{S} \beta_{k,s,j,t} \leq l_{j,t} && \forall k, j, t \\
& q_{j,s,t} \geq \beta_{k,s,j,t} && \forall k, s, j, t \\
& fu_{i,t}, pu_{i,t} \geq 0 && \forall i, t \\
& \beta_{s,k,t}, q_{j,s,t}, p_{k,j,t}, l_{j,t} \in \{0, 1\} && \forall i, s, k, m, j, t
\end{aligned}$$

By solving this model, we obtain the optimal distribution planning and the optimal allocation of customers to vehicles and of vehicles to seeds. The routes are calculated by solving a traveling salesman problem at each period for each vehicle based on the allocation found with the GAP.

5.2 Sequential transportation-production heuristic

This heuristic is based on a decomposition of the global model in three submodels which are solved sequentially. First, a transportation model for the finished products is solved. Then a production model is formulated and solved based on the solution of the finished product transportation model. Finally a distribution model for the shared resources is formulated and solved by taking into account the constraints resulting from the production planning and the finished product transportation model. Note that we could have solved those three submodels in the reverse way: First the transportation model for the shared resources then the production model and lastly the transportation model for the finished product. This is due to the fact that the two transportation models (shared resources and finished product) are perfectly symmetric.

5.2.1 The transportation decisions for finished product

The transportation decisions for the finished product determine the quantity of finished product as well as the routes of the various vehicles needed to satisfy the demand of the various customers. The model to solve is:

$$\begin{aligned}
\min\ & \sum_{j=1}^{J}\sum_{t=1}^{T} SCT_j * l_{j,t} \\
& + \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} HFC * fc_{i,k,t} \\
& + \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{m=1}^{K}\sum_{t=1}^{T} TC * length_{k,m} * f_{j,k,m,t} \\
\text{subject to} & \\
& (10), (12), (13), (15), (16), (17), (18), (19), (20), (30) \\
& z_{i,k,j,t} \geq 0 && \forall i,k,j,t \\
& fc_{i,k,t} \geq 0 && \forall i,k,t \\
& p_{k,j,t}, l_{j,t}, f_{i,k,m,t} \in \{0,1\} && \forall i,k,m,j,t
\end{aligned}$$

In order to solve this transportation model, we apply the same heuristic as the one used for the sequential production-transportation heuristic: we formulate a general assignment problem followed by a TSP for each vehicle at each period.

The distribution planning obtained for the finished product ($\overline{z_{i,k,j,t}}$) is recorded and will be used as data in the production planning subproblem.

5.2.2 The production planning

The production planning subproblem computes the quantity to produce at each period as well as the setup run in order to satisfy the distribution planning decisions.

The production planning problem optimizes (33) under the constraints:

$$x_{i,t} + fu_{i,t-1} = \sum_{k=1}^{K} \sum_{j=1}^{J} \overline{z_{i,k,j,t}} + fu_{i,t} \qquad \forall i,t \qquad (41)$$

$$x_{i,t} \leq M \times y_{i,t} \qquad \forall i,t \qquad (42)$$

$$x_{i,t}, fu_{i,t} \geq 0 \qquad \forall i,t \qquad (43)$$

$$y_{i,t} \in \{0,1\} \qquad \forall i,k,m,j,t \qquad (44)$$

This uncapacitated lot sizing model is solved using the same methodology as the one explained in Section 5.1.1.

5.2.3 The transportation decisions for the shared resources

At this decision level, it is necessary to determine the quantity of shared resources to collect at the various customer sites in order to satisfy the production planning decisions and the finished product transportation decisions.

As the quantity of shared resources to pick up at the customer site is restricted by the amount of finished product delivered (by constraint (14)), the transportation model for the shared resources reduces to a feasibility problem where a combination of linear inequalities have to be solved.

$$pc_{i,k,t-1} + D_{i,k,t-\delta_{i,k}} = pc_{i,k,t} + \sum_{j=1}^{J} w_{i,k,j,t} \qquad \forall i,k,t \qquad (45)$$

$$pu_{i,t-1} + \sum_{k=1}^{K} \sum_{j=1}^{J} w_{i,k,j,t-1} = pu_{i,t} + \overline{x_{i,t}} \qquad \forall i,t \qquad (46)$$

$$\sum_{i=1}^{I} \sum_{j=1}^{J} w_{i,k,j,t} = \sum_{i=1}^{I} \sum_{j=1}^{J} \overline{z_{i,k,j,t}} \qquad \forall k,t \qquad (47)$$

$$pu_{i,t} \geq 0 \qquad \forall i,t \qquad (48)$$

$$pc_{i,k,t} \geq 0 \qquad \forall i,k,t \qquad (49)$$

$$w_{i,k,j,t} \geq 0 \qquad \forall i,k,j,t \qquad (50)$$

$$pc_{i,k,t} \geq 0 \qquad \forall i,k,t \qquad (51)$$

5.3 An integrated production and distribution heuristic

In this section, we present a heuristic which is not based on a decomposition approach of the global model in submodels but which directly solve the global model in which a relaxed routing model is used. This heuristic gives a higher level of integration. It allows to take transportation considerations into account in the production planning problem.

Our integrated model (see Section 3.3) is difficult to solve due to the transportation decisions. Therefore, we approximate our transportation decision in the integrated model by using the adapted Fisher and Jaikumar heuristic for the VRP (see Section 5.1.3). As we are using the same approximation for the vehicle routing decisions as in the two decomposition heuristics, we are able to compare the performance of this heuristic with the two previous ones. This allows to highlight the value of integration.

The integrated GPA-Production planning model can be formulated as:

$$\min\ (1) + (2) + (4) + (5) + (6)$$

$$+ \sum_{k=1}^{K} \sum_{s=1}^{S} \sum_{j=1}^{J} \sum_{t=1}^{T} TC * \overline{length_{k,s}} * \beta_{s,k,j,t} \tag{52}$$

$$+ 2 * TC * \sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{t=1}^{T} length_{0,s} * q_{j,s,t} \tag{53}$$

subject to:

$$(7), (8), (10), (11), (12), (13), (14), (15), (16), (21), (22), (23), (27), (30), (32)$$

$$pu_{i,t-1} + puadd_i + \sum_{k=1}^{K} \sum_{j=1}^{J} w_{i,k,j,t-1} = pu_{i,t} + x_{i,t} \qquad \forall i, t \tag{54}$$

$$p_{k,j,t} = \sum_{s=1}^{S} \beta_{s,k,j,t} \qquad \forall k, j, t \tag{55}$$

$$\sum_{s=1}^{S} \beta_{k,s,j,t} \leq l_{j,t} \qquad \forall k, j, t \tag{56}$$

$$\sum_{s=1}^{S} \sum_{j=1}^{J} \beta_{k,s,j,t} \leq 1 \qquad \forall k, t \tag{57}$$

$$q_{j,s,t} >= \beta_{k,s,j,t} \qquad \forall j, k, s, t \tag{58}$$

$$y_{i,t}, p_{k,j,t}, l_{j,t}, q_{j,s,t}, \beta_{k,s,j,t} \in \{0, 1\} \qquad \forall i, k, s, j, t \tag{59}$$

Once this integrated GAP-production problem is solved, we know which customer is served by which vehicle, the production level and setup run for each product at each period, and the distribution planning. The routing of the different vehicles is performed in a second phase by solving a Traveling Salesman Problem at each period for each vehicle. This TSP is solved using TSP solver Concorde [1, 2].

6 Computational experiments

In this section, we highlight the advantages and disadvantages of our three heuristics. In order to achieve this objective, we perform three experiments.

The first test consists in analyzing the quality of the solution obtained with our three heuristics. Therefore, we compare the performance of our heuristics against an optimal solution approach in different shared resources capacity situations. Our aim is to analyze the “decomposition cost”: the increase in production and distribution costs implied by using a less coordinated solution methodology. This test is performed on an instance of reduce size (25 customers, 2 types of product, 5 vehicles, 5 time periods).

The last two computational tests are performed on 16 instances of larger size composed of 100 customers, 5 types of product, 7 vehicles and 5 time periods. Those instances vary according to the demand and customer location variability and mean.

The aim of the second experiment is twofold: analyze the performance of our heuristics in terms of computational time and analyze the impact of demand and customer location fluctuation and mean on the heuristics performance. This test is achieved in situation of scared and excess shared resource capacity.

The last computational analysis details the impact of variation in production and distribution costs on the performance of the heuristics. The goal is to analyze the impact of relative changes on the performance of our heuristic. This test is achieved with excess shared resource capacity.

Our computational experiments are obtained with commercial MIP solver XPRESS-MP and with TSP solver Concorde [1, 2].

6.1 Problem instances

The geographical position of customers is considered as following a normal distribution whereas the demand of customers follows a Gamma distribution.

The size of the different instances used varies according to the computational tests performed. For the first experiment, we use an instance of reduced size composed of 25 customers, 2 types of product, 5 vehicles and 5 time periods. For the two other tests, we have used 16 different instances composed each of 100 customers, 5 products, 7 vehicles and 5 time periods. Those instances differ on the type of demand (low/high variance, low/ high mean) and on the geographical position of the customer (low/high variance, low/high mean).

The various experiments are realized in situation where the amount of shared resources available in the system varies (scarce/excess capacity). More details on the dataset are given in the appendix.

	Int. GPA-Prod. plan.	Seq. transp.-prod. heur.	Seq. prod.-transp. heur.
Lot sizing cost	0%	0%	0%
Transportation cost	0%	0%	0.%
Total cost	0%	0%	0%

Table 1: Performance test: excess shared resources capacity

	Int. GPA-Prod. plan.	Seq. transp.-prod. heur.	Seq. prod.-transp. heur.
Lot sizing cost	0%	28.12%	28.12%
Transportation cost	0.2%	73.86%	73.86%
Total cost	0.02%	72.14%	72.14%

Table 2: Performance test: scarce shared resources capacity

6.2 Computational results

6.2.1 Quality of the heuristic solutions

Our aim in this section is to evaluate the performance of our three heuristics compared to an optimal solution approach. By optimal solution approach, we mean the solution obtained by solving our integrated model with a branch-and-cut technique where subtour elimination constraints are added as cuts at each integer node of the branch-and-bound tree. Therefore, we have tested the three heuristics and the optimal solution approach in situations of scarce and excess capacity. The production and distribution objective costs were fixed such that the importance of production cost compared to distribution cost were more or less the same. Due to the complexity of solving the global model to optimality [23], the various tests were performed on an instance of reduced size (25 customers, 2 types of products, 5 vehicles). The results (see Table 1 and Table 2) are given in terms of percentage of variation of cost (lot sizing cost and transportation cost) between the heuristics and the optimal solution procedure.

Two of the heuristics are based on a decomposition approach. This leads to a risk of infeasibility when there are not enough shared resources in the system (see Table 2). To avoid infeasibility problems, businesses need to rent extra capacity at an expensive cost. This explains the poor performance of those heuristics in that case.

In the case of the production-transportation heuristic, the stock minimal property of the production model implies that production takes place as late as possible (no production capacity limit). This production planning is optimal for the production lot sizing decisions but can be infeasible when shared resources limit are considered. This leads to more shared resources in the system. The additional amount of shared resources needed is determined when solving the transportation problem. For the transportation decisions, there is a coordination between the pick up of shared resources and delivery of finished products. This allows to optimize the transportation decisions.

For the transportation-production heuristics, the production decisions are taken considering the transportation decisions for the finished product as fixed. Nevertheless, the production planning does not consider the shared resources decisions which can lead to infeasibility problems. In this heuristic, we have two types of infeasibility that can occur. As production level and transportation quantities have been fixed before the shared resources decision, shared resources can be scarce at the production site as well as at the customer site (see Section 5.2.3).

Both of our decomposition heuristics give roughly the same result. This can be explained by the assumptions made on the model as well as the structure of the different submodels. Regarding the production-transportation heuristic, we know that the production will take place as late as possible (see Section 5.1.1 for more details). Therefore, when the transportation submodel is solved (with the production variables fixed), the transportation of finished goods will be performed as late as possible. As there is no transportation lead time, no demand backlogging and no transportation capacity limit, the production and transportation of finished goods will be done simultaneously. Concerning the transportation-production heuristic, the same reasoning can be applied. The structure of the transportation submodel implies that the amount transported takes place as late as possible. Therefore, when the production level is calculated with the Wagner-Whitin production submodel (with the amount transported fixed by the previous submodel), we produce exactly what needs to be sent. Note that additional feasibility problem could have appeared if the transportation lead time and/or transportation capacity was constraining.

The integrated GAP-production heuristic does not have these infeasibility problems. This leads to better performance compared to our decomposition heuristics. We can also observe that the performance of our integrated heuristic is very close to the performance of the optimal solution approach.

6.2.2 Performance of the heuristics

In this section, we report results of tests of our three heuristics on 16 instances of larger size composed of 100 customers, 5 types of product, 7 vehicles and 5 time periods. The three heuristics were run for 1800 seconds at most and with different shared resource capacity limits (excess/scarce capacity limit). We report results in terms of percentage of variation in production and distribution cost between the decomposition heuristics and the integrated heuristic.

The optimal solution approach was unable to even provide a feasible solution to the global production and distribution model for any instance in the given time.

Cost analysis

- Unlimited shared resources capacity

  In the case of unlimited shared resources capacity in the system, the three heuristics give the same lot sizing and transportation cost for the 16 instances. As there is enough shared resources in the system, the impact of no coordination between the production and distribution department is absorbed by the excess amount of shared resources in stock.

- Limited shared resources capacity

  In the case of limited shared resource capacity (see Tables 3 and 4), we observe the same behavior as in the first test. The two decomposition heuristics perform badly due to feasibility problems. In this scenario, extra capacity has to be rented at an expensive cost which leads to poor results.

  From Tables 3 and 4, we observe that integration of decisions allows to gain in production as well as in transportation cost. Moreover, the value of integration is greater when the demand of customers has low variability (data sets 9 to 16 (see the appendix for more details)) and the demand's mean is low. Nevertheless, the demand's mean do not seem to have an impact as strong as the variability of the demand. In addition, the variability and mean of customer locations does not seem to have an impact on the value of integration. In conclusion, the combination of low demand variability and mean allows to benefit the most from integration of production and distribution decisions.

	Instances	Seq. transp.-prod. heur.	Seq. prod.-transp. heur.
Lot sizing cost	1	28.34%	28.39%
	2	28.61%	28.61%
	3	27.41%	28.33%
	4	28.25%	28.52%
	5	18.12%	18.16%
	6	18.07%	18.27%
	7	15.24%	18.16%
	8	18.31%	18.33%
	9	42.24%	42.25%
	10	42.24%	42.24%
	11	42.13%	42.22%
	13	42.08%	42.24%
	13	46.33%	46.33%
	14	46.48%	46.48%
	15	46.23%	46.23%
	16	46.31%	46.31%

Table 3: Lot sizing Cost analysis: scarce shared resources capacity-results are reported in terms of percentage of variation in production and distribution cost between the decomposition heuristics and the integrated heuristic.

	Instances	Seq. transp.- prod. heur.	Seq. prod.- transp. heur.
Transportation cost	1	50.31%	50.37%
	2	50.38%	50.38%
	3	49.92%	50.26%
	4	50.03%	50.36%
	5	33.57%	33.64%
	6	33.32%	33.63%
	7	29.00%	33.57%
	8	33.58%	33.61%
	9	79.21%	79.21%
	10	79.22%	79.22%
	11	79.12%	79.19%
	12	79.10%	79.21%
	13	75.33%	75.33%
	14	75.35%	75.35%
	15	75.25%	75.35%
	16	75.33%	75.33%

Table 4: Transportation Cost analysis: scarce shared resources capacity-results are reported in terms of percentage of variation in production and distribution cost between the decomposition heuristics and the integrated heuristic.

Computational time analysis The three heuristics were run for a maximum of 1800 seconds on the 16 instances in both limited and unlimited shared resource capacity situations. Our aim here is to compare the time performance of our heuristics. As the most time consuming phase of our heuristics was, in each case, the subproblem using the general assignment heuristic methodology, we report only the time performance of this phase. Indeed, the subproblem using the general assignment heuristic is accounting for 98% of the computational time. Therefore, we present performance graphs (see Figures 2 and 3) that report, for each solution method, the gap between each integer solution found in 1800 seconds and the lower bound for the corresponding heuristic expressed in percentage. As the subproblem using the general assignment heuristic differs for each of our heuristics, the lower bound used to calculate our gap is different. Therefore, the performance graphs can only be used to analyze the time performance of our heuristics and not the quality of the solution obtained (which was the aim of the previous computational experiments). Note that the x axes is represented in log scale.

In the case of unlimited shared resources in the system, we observe that the production-transportation heuristic is less time consuming than the two other heuristics. First of all, the production-transportation heuristic finds less integer solutions than the two previous heuristics and secondly the "best" integer solution, which is the same for the three heuristics, is found earlier than for the other heuristics. Figure 2 presents a typical performance graph of the three heuristics in the case of unlimited shared resources capacity.

In the case of limited shared resources, the transportation-production heuristic is the more time consuming while giving worse results than the integrated heuristic and slightly better result than the production-transportation heuristic. Figure 3 presents a typical performance graph of the three heuristics in the case of limited shared resources capacity.

Even though the transportation-production heuristic performs a little better than the production-transportation heuristic, the time performance is a lot worse and does not justify the gain obtained (on average a 0.53% improvement in total cost with the transportation-production heuristic compared with the production-transportation heuristic). The production-transportation heuristic is less time consuming than the integrated heuristic but gives worse result in terms of total production and distribution costs. Nevertheless, the first integer solution for our integrated heuristic is found in maximum 52 seconds and is better than the production-transportation heuristic solution because it does not use extra shared resources capacity.

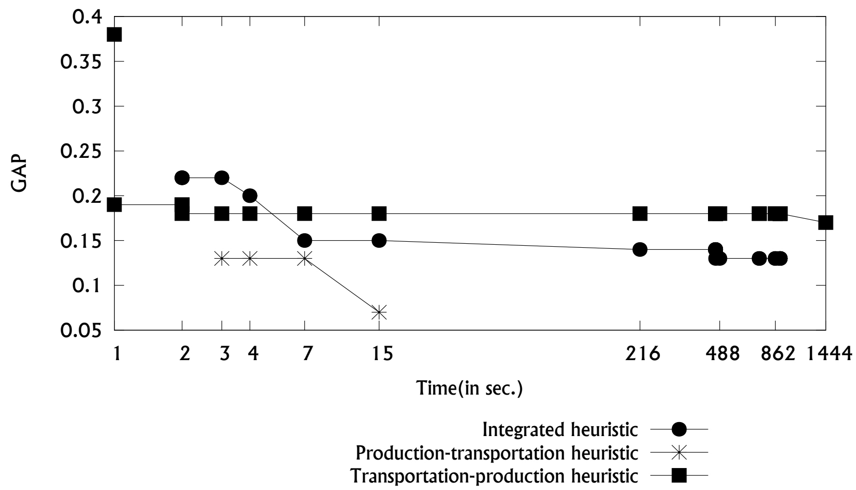

Figure 2: Performance profile in the case of unlimited shared resource capacity - The gap represents the difference between each integer solution found in 1800 seconds and the lower bound for the corresponding heuristic expressed in percentage

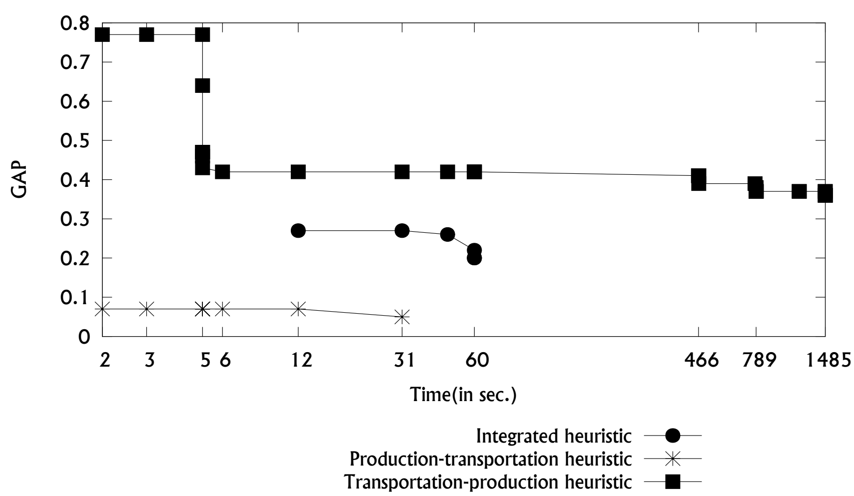

Figure 3: Performance profile in the case of limited shared resource capacity - The gap represents the difference between each integer solution found in 1800 seconds and the lower bound for the corresponding heuristic expressed in percentage

6.2.3 Sensitivity analysis

We have performed a sensitivity analysis to assess the impact of the production and distribution objective function costs on the performance of the three heuristics. We have used the same instances as the one in the previous test with excess shared resources.

Our aim in those tests is to consider changes in distribution costs relatively to production costs. Those changes can easily occur in business whenever fuel price, taxes, ... change.

For the transportation-production heuristic, there is no impact on the production and distribution planning when there are changes in production and distribution objective function costs. This is due to the fact that the objective function of each submodel is composed only of production or distribution costs.

In the case of the production-transportation heuristic, the production and distribution planning are invariant to changes in the production and distribution objective function costs. The production model considers only production costs which explains why there is no impact on the production planning. Even though the transportation problem contains production cost (storage cost at the manufacturing site), it is preferable to produce and directly send products to customers. This can be explained by the structure of our model. It would have been interesting to store finish product at the production facility site if there was a production capacity limit. In our model, we have omitted this limit which implies that whatever relative difference between production and distribution cost, it is never interesting to store at the manufacturing site.

For the integrated heuristic, changes in production and distribution cost have an impact on the production and distribution planning. When distribution costs are greater than production costs, the production planning includes more setups. This is due to the fact that as storage cost is important at the customer site, the demand is satisfied as much as possible on time whereas when distribution costs are less than production costs, the demand is satisfied on stock. In the later case, there will be less production setup and more storage at the customer site. Nevertheless, the impact of those changes on the performance (better production and distribution costs) of the integrated heuristic compared to the decomposition heuristic are negligible.

7 Conclusion

Nowadays, coordination of all functional areas is fundamental in order to improve the efficiency of the supply chain. With the development in information and technological tools, it is now possible to develop decision tools which help manager to coordinate decisions at all level in the supply chain. Our aim in this article is to analyze the advantage of coordination between the production and the distribution department when resources are shared between those two functional areas. We focus our study on three decisions: lot-sizing decisions at the production and distribution level and vehicle routing decisions. Therefore, we develop a global production/distribution model and solve this model with three heuristics. Two of those heuristics are based on a decomposition approach of the global model in production and transportation submodels. The third heuristic offers a higher level of integration by considering transportation decisions when solving the production problem. In all of those heuristics, the transportation decisions are approximated based on Fisher and Jaikumar's approach. Computational tests show that the performance of the heuristics depends on the amount of shared resources in the system, on the variability of customer demand and not on the weight of production cost against distribution cost. In addition, we show that the three heuristics allow to solve instances of larger size than an optimal solution approach.

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Appendix

Description of the dataset

Instance	Demand variability	Customer location variability	Demand mean	Customer location mean
1	high	high	low	low
2	high	low	low	low
3	high	high	low	high
4	high	low	low	high
5	high	high	high	low
6	high	low	high	low
7	high	high	high	high
8	high	low	high	high
9	low	high	high	low
10	low	low	high	low
11	low	high	high	high
12	low	low	high	high
13	low	high	low	low
14	low	low	low	low
15	low	high	low	high
16	low	low	low	high

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